7th Asian Workshop on Symmetric Key Cryptography (ASK2017)
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Innovative R&D by NTT

Some Improvements of Non-Blackbox Cube Attacks

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Today's Talk

1. Cube Attacks on Non-Blackbox Polynomials.
   - Proposed at CRYPTO 2017.
   - New generic tools for the cube attack.
2. Improvement 1.
   - Longer distinguisher is found when inactive bits are 0.
   - In detail, ePrint/2017/306.
3. Improvement 2.
   - Reduce the time complexity by exploiting low degree property of superpoly.
   - In detail, ePrint/2017/1063.

Cube Attacks on Non-Blackbox Polynomials (from CRYPTO2017)

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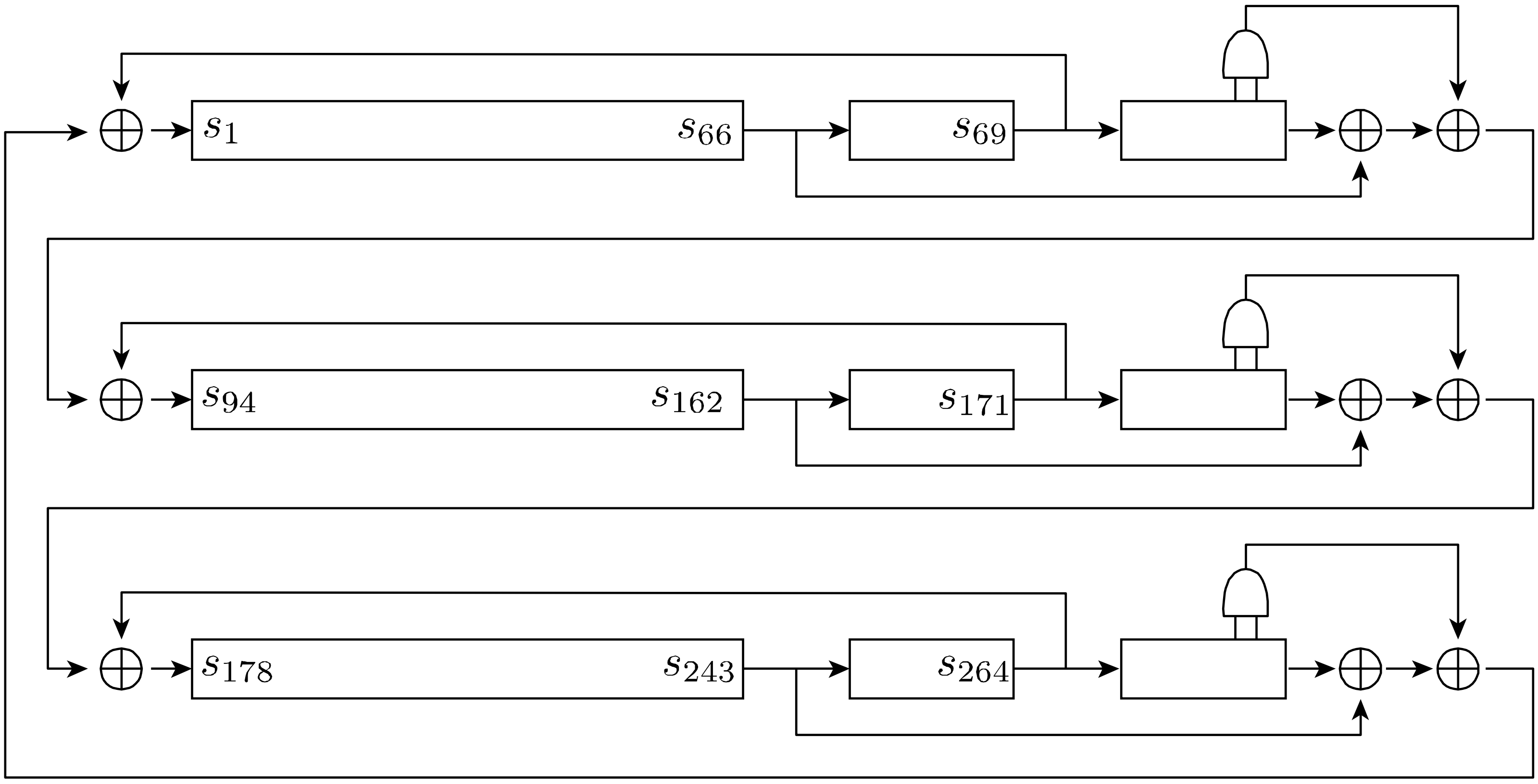
Stream ciphers

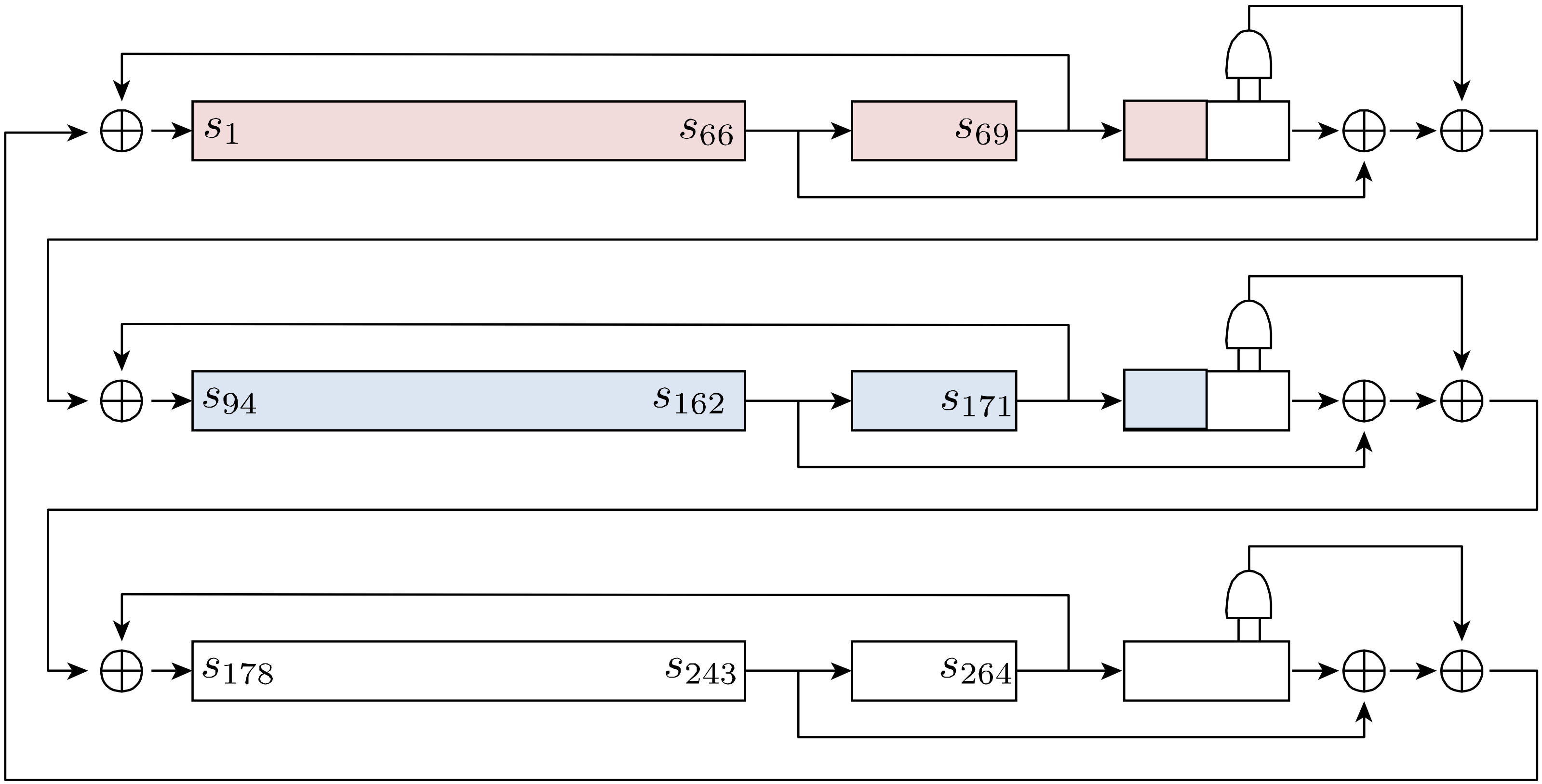
- Consists of two parts
  - Key initialization.
    - Secret key and public IV are loaded to the internal state.
    - Execute the update function iteratively w/o output of key-stream sequence.
  - Key-stream generation.
    - Update function outputs key-stream sequence.

Example of Trivium : Internal State



state size = 288 bits

Example of Trivium : Key initialization



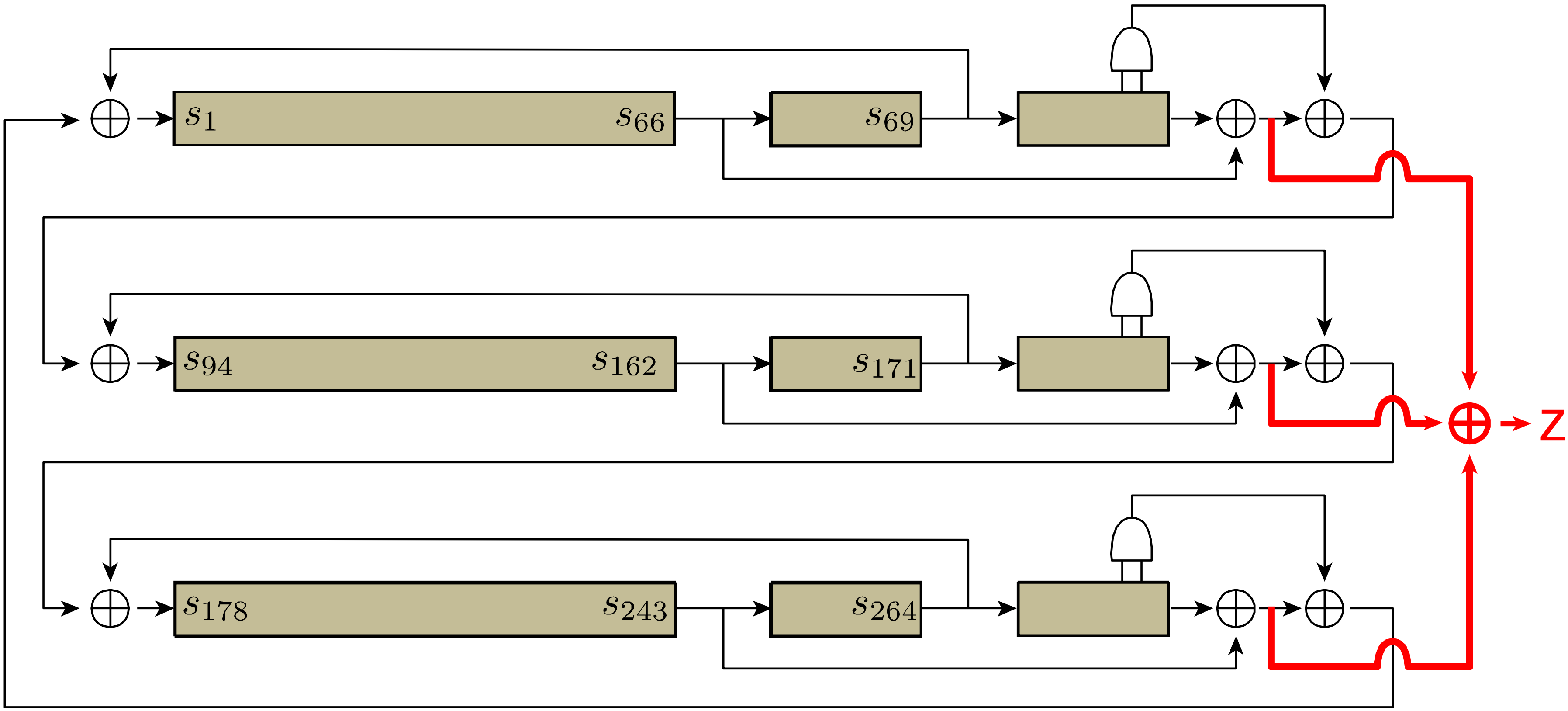
80-bit secret key

80-bit initialization vector

state size = 288 bits

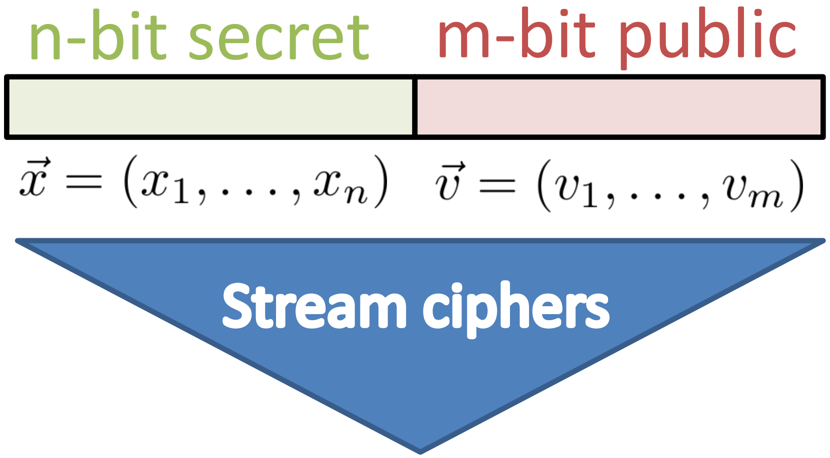
initialization = 1152 rounds

Example of Trivium : Output key stream



1 update function outputs 1-bit key stream.

Stream ciphers



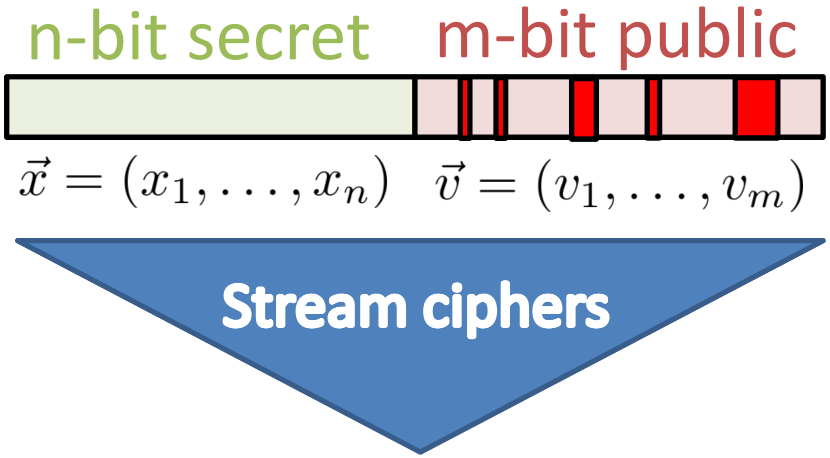
- $\vec{x}$ is n-bit secret variable.
- $\vec{v}$ is m-bit public variable.
- z is the first bit of the key stream.

$$z = f(\vec{x}, \vec{v}) = \bigoplus_{\vec{u} \in \mathbb{F}_2^{n+m}} a_{\vec{u}}^{f} \cdot \vec{x}^{\vec{u}} \cdot \vec{v}^{\vec{v}}$$

ex) $z = x_1 x_2 \oplus x_1 v_1 \oplus v_2 v_3$

Idea of the cube attack [DS09]

$$t_I = v_{i_1} \times \cdots \times v_{i_{|I|}}$$



- Let $I = \{i_1, \ldots, i_{|I|}\}$ be the indices of active bits.
- Let C_I be a set of $2^{|I|}$ values where v_i $(i \in I)$ is active.

$$z = f(\vec{x}, \vec{v}) = t_I \cdot p_I(\vec{x}, \vec{v}) + q_I(\vec{x}, \vec{v})$$

$$\bigoplus_{v \in C_I} z = p_I(\vec{x}, \vec{v})$$

Attackers recover secret variable $\vec{x}$ by analyzing $p_I(\vec{x}, \vec{v})$.

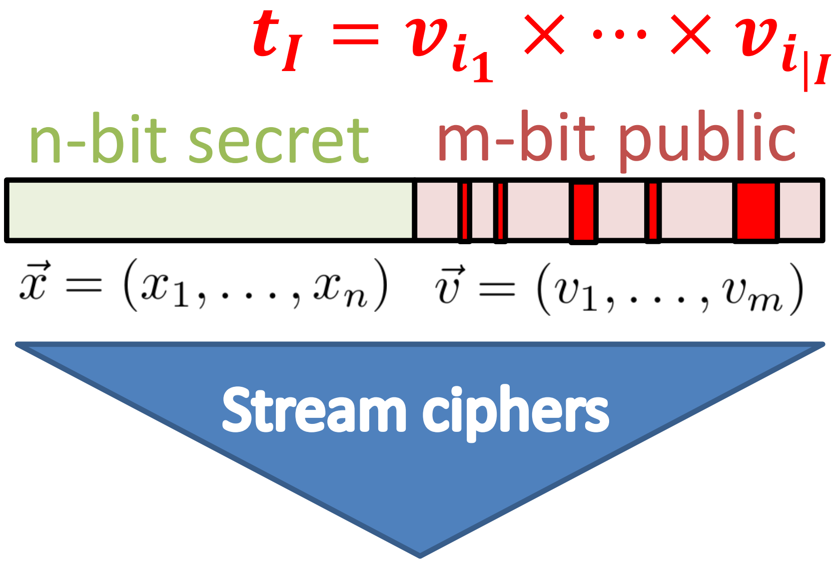
Concrete example

$$
\begin{aligned}
&f(v_1, v_2, v_3, x_1, x_2) \\
&= v_1 v_2 v_3 + v_1 v_2 x_1 + v_2 x_1 x_2 + v_1 v_2 + v_2 + v_3 x_2 + x_2 + 1 \\
&= v_1 v_2 (v_3 + x_1 + 1) + (v_2 x_1 x_2 + v_3 x_2 + v_2 + x_2 + 1)
\end{aligned}
$$

$$
\begin{cases}
t_I = v_1 v_2 \\
p_I(\vec{x}) = v_3 + x_1 + 1 \\
q_I(\vec{x}) = v_2 x_1 x_2 + v_3 x_2 + v_2 + x_2 + 1
\end{cases}
$$

$$
\bigoplus_{(v_1, v_2) \in \{0,1\}^2} f(\vec{v}, \vec{x}) = v_3 + x_1 + 1
$$

Unfortunately…



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$$z = f(\vec{x}, \vec{v}) = t_I \cdot p_I(\vec{x}, \vec{v}) + q_I(\vec{x}, \vec{v})$$

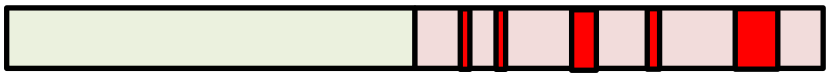
We cannot decompose $f(\vec{x}, \vec{v})$ because real stream cipher is complicated.

Experimental balckbox analysis

- How to recover $p_I(\vec{x}, \vec{v})$.
  1. Assume that p_I is linear function.
  2. Randomly choose $\vec{x}$.
     iteratively compute $\bigoplus_{\vec{v} \in C_I} f(\vec{x}, \vec{v}) = p_I(\vec{x}, \vec{v})$.
  3. Execute linearly test on many $\vec{x}$.
     Recover p_I under the assumption that it's linear.

- Drawback
  - The cube size is limited in the range of experimental, e.g., $|C_I| \leq 40$.

Motivation



$\vec{x} = (x_1, \ldots, x_n)$ $\vec{v} = (v_1, \ldots, v_m)$

$$z = f(\vec{x}, \vec{v})$$

Experimental cube attack

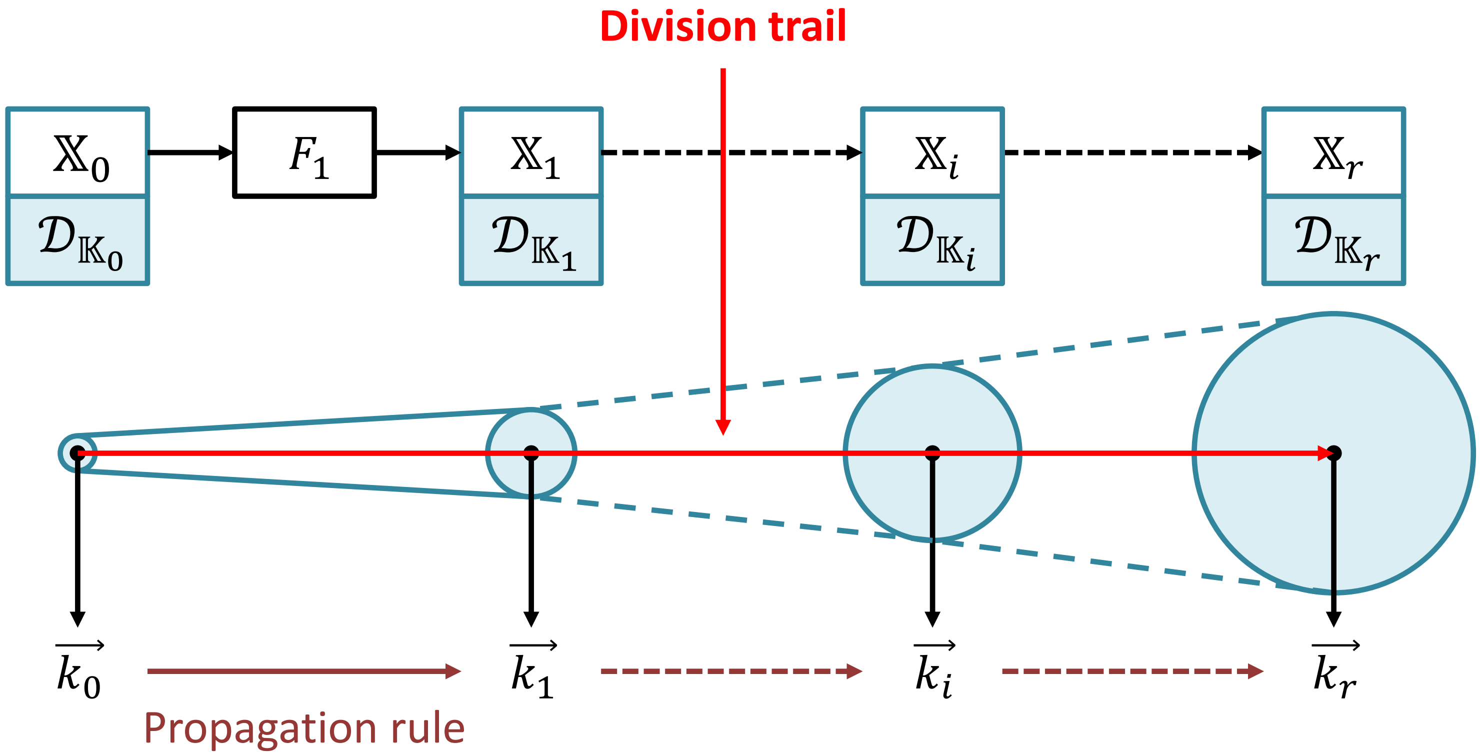
- Iterate linearly test experimentally.
- Recover the ANF of superpoly in real.

Theoretical cube attack

- Analyze the structure of superpoly.
- Evaluate the ub to recover its ANF.

We use the ***division property*** as a tool to analyze the sturcture of the superpoly.

Division Property



If there is NOT division trail $\vec{k}_0 \xrightarrow{f = F_r \circ \cdots \circ F_1} \vec{1}$,
the output of the Boolean function f is balanced.

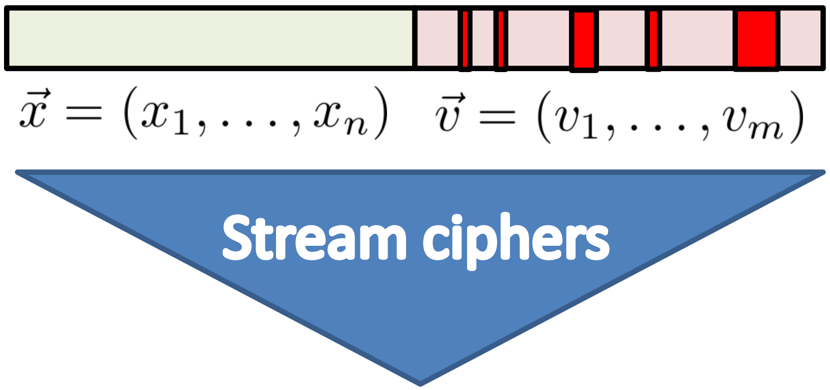
※ $\vec{v}^{\vec{k}_0} = t_I$.

How to analyze division trails?

- Programming from scratch.
  - Depth/Breadth First Search.
- CP-based approaches.
  - Mixed Integer Linear Programming.
  - SAT solver.
  - Constraint Programming.

Zero-sum distinguisher

$t_I = v_{i_1} \times \cdots \times v_{i_{|I|}}$



$\vec{x} = (x_1, \ldots, x_n)$ $\vec{v} = (v_1, \ldots, v_m)$

$$z = f(\vec{x}, \vec{v})$$

Division property

$$(\vec{0}, \vec{k}), \quad \vec{v}^{\vec{k}} = t_I$$

No division trail.

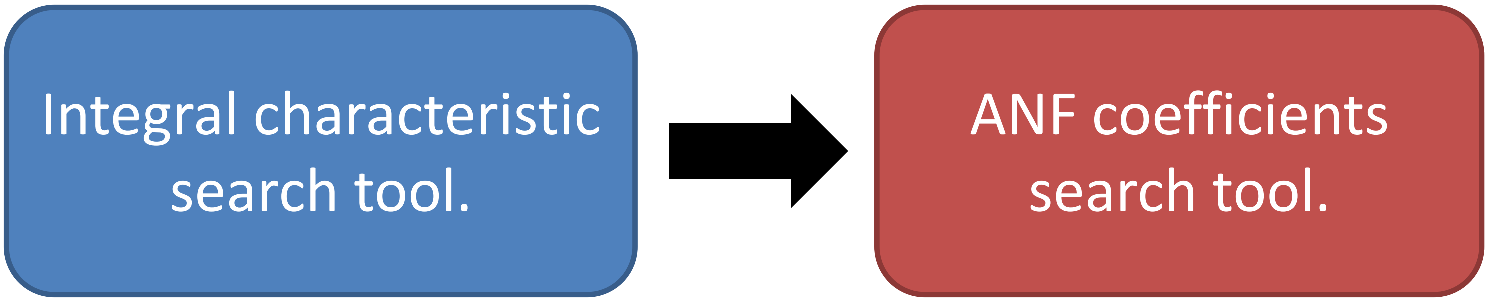
$$1$$

$$\bigoplus_{v \in C_I} f(\vec{x}, \vec{v}) = 0$$

Zero-sum distinguisher is trivially application.

How to recover the ANF.

- The role of division property.



- We revisit what the division property can do.

What division property can do

- Assuming there is NOT trail $\vec{k} \xrightarrow{f(\vec{x})} 1$,

$$\bigoplus_{C_I} f(\vec{x}) = p(\vec{x}) = \bigoplus_{\vec{u} \in \mathbb{F}_2^n | \vec{u} \succeq \vec{k}} a_{\vec{u}}^f \cdot \vec{x}^{\vec{u} \oplus \vec{k}}$$

is always zero for any $\vec{x}$.

- In other words,
  - $a_{\vec{u}}^f$ is always 0 for any $\vec{u} \succcurlyeq \vec{k}$.
- Division property can be used to analyze ANF coefficients.

Extension to key recovery.

- Assuming there is NOT trail $(\vec{e}_j, \vec{k}) \xrightarrow{f(\vec{x},\vec{v})} 1$, $a^f_{\vec{u}}$ is always 0 for any $\vec{u} \succcurlyeq (\vec{e}_j || \vec{k})$.
- Then,

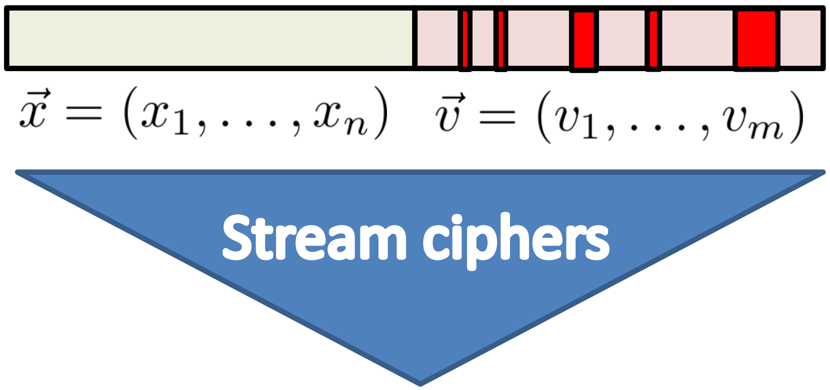
$$\bigoplus_{C_I} f(\vec{x}, \vec{v}) = p(\vec{x}, \vec{v}) = \bigoplus_{\vec{u} \in \mathbb{F}_2^{n+m} | \vec{u} \succeq (\vec{0} || \vec{k}_I)} a^f_{\vec{u}} \cdot (\vec{x} || \vec{v})^{\vec{u} \oplus (\vec{0} || \vec{k}_I)}$$

$$= \bigoplus_{\vec{u} \in \mathbb{F}_2^{n+m} | \vec{u} \succeq (\vec{0} || \vec{k}_I), u_j = 0} a^f_{\vec{u}} \cdot (\vec{x} || \vec{v})^{\vec{u} \oplus (\vec{0} || \vec{k}_I)}.$$

- The superpoly is independent of x_j becase $x_j^{u_j} = x_j^0 = 1$.

Summary of division property-based cube

$t_I = v_{i_1} \times \cdots \times v_{i_{|I|}}$



$\vec{x} = (x_1, \ldots, x_n)$ $\vec{v} = (v_1, \ldots, v_m)$

$z = f(\vec{x}, \vec{v})$

Division property

$(\vec{e}_j, \vec{k}), \quad \vec{v}^{\vec{k}} = t_I$

1

No division trail.

x_j is not involved to $\bigoplus_{v \in C_I} f(\vec{x}, \vec{v})$

By repeating this procedure,
we can distinguish which secret-key bits are involved.

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Applications.

Applications	Previous Best	New Best
Trivium	799	832
Grain128a	177	183
ACORN	503	704
Kreyvium	--	872

※Applications to Kreyvium are explained the full version (ePrint/2017/306)

1st Improvement.
Exploiting constant-0 cubes.
(from ePrint/2017/306)

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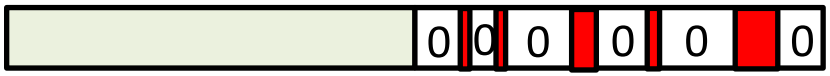
Willi Meier, FHNW, Switzerland

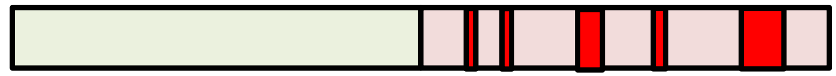
Motivation

We want to fill the gap from other works.



$\vec{x} = (x_1, \ldots, x_n)$ $\vec{v} = (v_1, \ldots, v_m)$

Non-active bits are always 0 in many previous cubes.



$\vec{x} = (x_1, \ldots, x_n)$ $\vec{v} = (v_1, \ldots, v_m)$

Non-active bits are any value in our cubes.

$$f(v_1, v_2, v_3, x_1, x_2)$$
$$= v_1 v_2 (v_3 + x_1 + v_3 x_2 + 1) + (v_2 x_1 x_2 + v_3 x_2 + v_2 + x_2 + 1)$$

$$p(v_3, x_1, x_2) = v_3 + x_1 + v_3 x_2 + 1$$
$$p(0, x_1, x_2) = x_1 + 1$$

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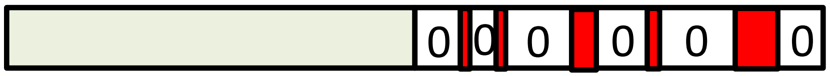
Motivation

We want to fill the gap from other works.



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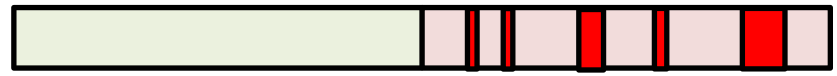
Non-active bits are always 0 in many previous cubes.



$\vec{x} = (x_1, \ldots, x_n)$ $\vec{v} = (v_1, \ldots, v_m)$

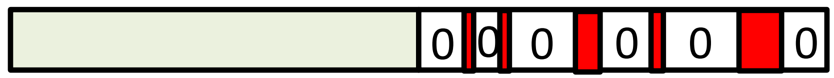
Non-active bits are any value in our cubes.

- 0-constant cubes bring more powerful attack generally.
- Liu's cube (at CRYPTO17) also uses 0-constant cube.

We need a new technique to exploit 0-constant cube with the division property.

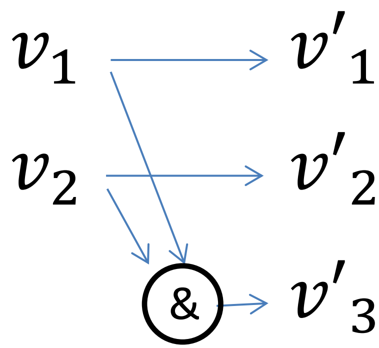
Exploiting the constant 0

- Non-cube bits are 0.



$\vec{x} = (x_1, \ldots, x_n)$ $\vec{v} = (v_1, \ldots, v_m)$

  - If non-cube bit is fixed to 0, the propagation of the division property is restricted.

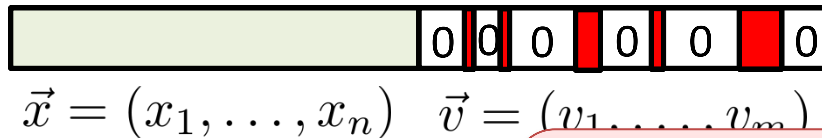


(0,0) → (0,0,0)
(1,0) → (1,0,0), (0,0,1)
(0,1) → (0,1,0), (0,0,1)
(1,1) → (1,1,0), (0,0,1)

※Similar technique was already used by Sun et al's work in the context of the integral distinguisher (ePrint/2016/1101).

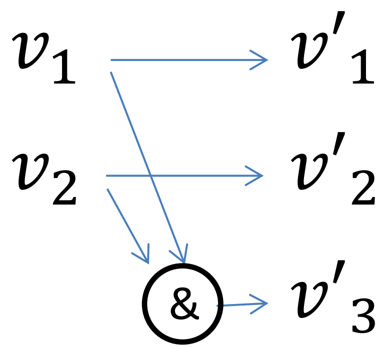
Exploiting the constant 0

- Non-cube bits are 0.



  - If non-cube bit is fixed t
    the division property is

Condition : $v_2 = 0$



(0,0) → (0,0,0)

(1,0) → (1,0,0), ~~(0,0,1)~~

~~(0,1) → (0,1,0), (0,0,1)~~

~~(1,1) → (1,1,0), (0,0,1)~~

impossible propagation

※Similar technique was already used by Sun et al's work in the context of the integral distinguisher (ePrint/2016/1101).

Summary of distinguishing attacks.

Applications	rounds	cube size	type	method
Trivium	837	37	zero sum	Liu & ours
	838	38	zero sum	ours
	842	37	biased sum	experimental (Liu)
Kreyvium	872	61	zero sum	Liu & ours
	873	62	zero sum	ours

- We can revisit Meicheng Liu's result.
- We can improve the zero-sum distinguisher on Trivium and Kreyvium from Liu's result.
- We haven't tried experimental approaches.
  - There is the possibility 38-dimensinal cube derives stronger biased sum distinguisher.

Comparison between Liu's result

	Liu's algorithm	Division property	Comment
Complexity	**WIN**	LOSE	We need to ask for solver's help to evaluate the division trails.
Accuracy	LOSE	**WIN** (w/ improved technique.)	I find some instances that division property is better than Liu's algorithm.
Flexibility	LOSE	**WIN**	Division property is applicable to arbitrary ciphers.

- Recommendation.
  - If the solver can stop, division property is better.
  - Otherwise, e.g., the state size is too large, we have to use Liu's algorithm.

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2^{nd} *Improvement.* Exploiting Low Degree Property of Superpoly *(ePrint/2017/1063)*

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Yosuke Todo, NTT Secure Platform Laboratories, Japan
Chaoyun Li, KU Leuven, Belgium
Takanori Isobe, University of Hyogo, Japan
Willi Meier, FHNW, Switzerland

Motivation

Experimental cube attack.

- Superpoly is assumed as linear or quadratic.
  - Experimental cube recovers superpoly efficiently by exploiting this low degree property.

Take one step further!!

- We also exploit this low-degree property with the division property.
  - The upper bound of the degree on superpoly is estimated.
  - The time complexity is more reduced.

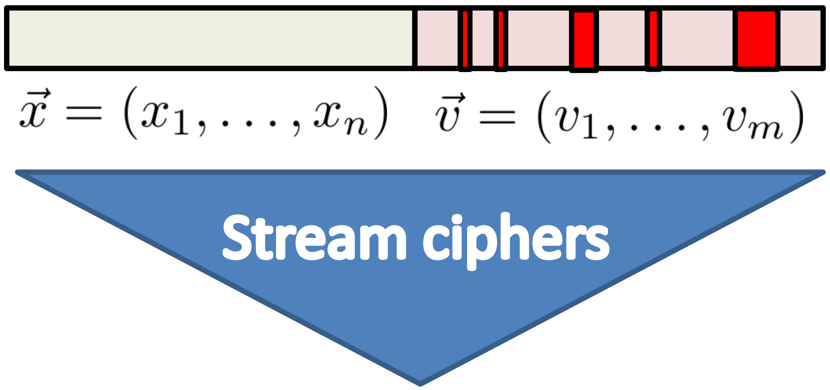
If the superpoly is low degree,...

- If the degree is at most d.
  - We don't need to evaluate the ANF coefficients whose degree of monomials is more than d.
  - The time complexity is reduced from

$$2^{|I|+|J|} \text{ to } 2^{|I|} \times \sum_{i=0}^{d} \binom{|J|}{i}$$

How degree is evaluated?

$t_I = v_{i_1} \times \cdots \times v_{i_{|I|}}$



$\vec{x} = (x_1, \ldots, x_n)$ $\vec{v} = (v_1, \ldots, v_m)$

$z = f(\vec{x}, \vec{v})$

Division property

$(\vec{\ell}, \vec{k}), \quad \vec{v}^{\vec{k}} = t_I$

1

No division trail.

under this condition

$\text{maximize} \sum_{j \in J} \ell_j$

This maximum value corresponds the upper bound of the algebraic degree of the superpoly.

Applications and results

Applications	rounds	cube size	|J|	time	ref.
Trivium	832	72	5	2^{77}	crypto17
	839	78	1	2^{79}	ePrint/2017/1063
Kreyvium	872	85	39	2^{124}	ePrint/2017/306
	888	102	36	2^{111.38}	ePrint/2017/1063

- Focus on 888-round attack on Kreyvium.
  - The number of involved secret variables is 36.
  - Previous estimations requires 2^{138} complexity.
  - However, since the degree of superpoly is at most 2, we can dramatically reduce the complexity.

Conclusion

- Division property based cube attacks
  - A new generic framework to evaluate the security against cube attacks.
  - It brings best key-recovery attacks against Trivium, Grain128a, ACORN, Kreyvium.
- Further improvements
  - Exploiting constant-0 cube brings more powerful superpoly recovery attacks.
  - Exploiting low degree property of the superpoly reduce the time complexity to recover the superpoly.